Robust Algorithms and EP theorems II

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Schedule of the talk

Certifying algorithms (even interesting for problem in P)

Polynomial certificates

NP is the class of decision problems with a polynomial certificate for the YES Instances.

For an instance I, a polynomial certificate is $C(I)$ a word of size polynomial in $|I|$ that can be checked in polynomially in $|I|$.

For a problem in NP, the theory only provides the existence of such a certificate (at least the execution scheme of the associated Turing Machine).

- For Hamilton path, the certificate is just some hamilton path.
- It is not always so simple (primality or geometric problems)

Geometric problems

Visibility Graph Recognition

Given a visibility graph G and a Hamiltonian circuit C, determine in polynomial time whether there is a simple polygon whose vertex visibility graph is G, and whose boundary corresponds to C.

Partial and Related Results

The problem is not even known to be in NP [O'R93], although it is for "pseudo-polygon" visibility graphs [OS97].

Good characterizations

$NP \cap co - NP$ polynomial certificate in both cases
Good characterizations

EP theorems, K. Cameron and J. Edmonds 1990

Definition

An EP (Existentially Polytime) theorem is a theorem in which each condition is polynomially testable.

For example

($\exists$ some certificate α) or ($\exists$ a certificate β) :..

Algorithm

For such a problem, we can hope to find as output one of the certificates of the theorem.

Particular case :

- When the 2 cases are exclusive, we come back to good characterizations.
- Jack's yesterday dreamed for more : an algorithmic proof which provides both certificates
  True for many examples (matching, flow . . .).

Robust algorithms, J. Spinrad 2002

For an NP-complete optimisation problem (ex : coloration), when considering a particular class $\mathcal{C}$ of graphs, a polynomial algorithm is called robust if it satisfies the following conditions :

Robust algorithms, J. Spinrad 2002

1. If the data belongs to the $\mathcal{C}$, the algorithm gives the good answer
2. Else :
   - Either the algorithm gives the good answer
   - or the algorithm answers that the input data does not belong the class $\mathcal{C}$ and provides a certificate of this fact.

- Important, if you cannot completely trust the data coming from an application (noise, errors ...)
- An algorithm only satisfying the first condition is called unrobust algorithm.
- Most of the algorithm designed for graph classes are unrobust, and therefore not so interesting if the class $\mathcal{C}$ is NP-complete to recognize.

- It is not always possible to derive a polynomial recognition algorithm for the class $\mathcal{C}$ from a robust algorithm operating on $\mathcal{C}$.
- In fact a robust algorithm provides a recognition algorithm for a class wider than $\mathcal{C}$. This class may depend on the choices made by the algorithm.

When working on optimisation with graph classes this notion of robust algorithm appears to be very interesting.

Unit Disk graphs

Definition

A graph is a unit disk graph if is the intersection graph of a set of unit disks in the plane

Bad news

Recognizing unit disk graphs is NP-hard, although not known to be in NP.

Good news

There is a robust algorithm to compute a maximum clique on unit disk graphs.

Using the geometry, transform the problem into maximum clique in co-bipartite graphs

Well covered graphs

Definition

A graph is well covered if every maximal independent set is a maximum independent set

Bad news 1

Recognizing well covered graphs is NP-complete, although is it easy to compute a maximum independent set on this class.

Bad news 2

There is no robust algorithm for independent set on well covered graphs unless P=NP.
Using a simple reduction from independent set.

Open problem

Visibility graphs

Find a robust algorithm for the max clique in visibility graphs.
Polynomial when the polygon is given using dynamic programming

EP theorems and robusts algorithms

Every robust algorithm corresponds to an EP theorem.
($\exists$ an optimal solution together with its certificate) or ($\exists$ a certificate showing that the input data does not belong to $\mathcal{C}$)

An EP theorem can be a guide for the search for a robust algorithm.

- Good characterisation versus Polynomial algorithm
- EP theorem versus Robust algorithm

- I prefer EP theorems to formulate things mathematically because of its symmetry between conditions
- But Robust algorithms are "natural" coming from applications
- Both ideas was published and quite ignored

Conclusions

2 interesting problems :

1. Jack's first question : polynomial to check in both answers is it equivalent to polynomial to compute ?
2. Jack's second question : EP theorems and algorithms
3. What are the general conditions, for a given problem, to derive a certificate easy to check ?

- As you may know, Jack is mostly interested in algorithmic proofs, and does not care about induction proofs.
- This lecture was just to show that Jack's ideas are closed to software engineering (certifying algorithms) and central in computer science, for the design of nice algorithms.

Bibliography

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Thanks for your attention ! !