Abstract:
k-dominating k-independent sets
Odile Favaron, Wednesday, Feb. 10 at 16:30

Let k be a positive integer. A subset S of the vertex set V of a graph G is *k-independent* if $\Delta(S) < k$ where $\Delta(S) = \max\{d_S(x) \mid x \in S\}$ and $d_S(x)$ is the number of neighbors in S of x. The subset S is *k-dominating* if $d_S(v) \geq k$ for each vertex $v \in V \setminus S$. The maximum cardinality of a k-independent set and the minimum cardinality of a k-dominating set are respectively denoted $\beta_k(G)$ and $\gamma_k(G)$. The case $k = 1$ corresponds to the usual independent (or stable) sets and dominating sets. Hence $\beta_1(G) = \beta(G)$, the independence number, and $\gamma_1(G) = \gamma(G)$, the domination number.

It is known that $\gamma(G) \leq \beta(G)$ for all graphs because every maximal (by inclusion) independent set is dominating. But for $k \geq 2$, a maximal k-independent set is not necessarily k-dominating and the comparison between β_k and γ_k is not so obvious. Fink and Jacobson, who introduced this generalization in 1984, conjectured that $\gamma_k(G) \leq \beta_k(G)$ for every k and every G. To prove this conjecture, it is sufficient to prove the existence of subsets which are both k-independent and k-dominating.

We present two proofs of this property. The first one [**?**] establishes the existence theorem by contradiction and then can be translated into a polytime algorithm to construct k-independent k-dominating sets. The second one [**?**] directly gives an algorithm of construction of such a set.

References

[1] O. Favaron, On a conjecture of Fink and Jacobson concerning k-domination and k-dependence, J. Comb. Theory Ser. B 39(1): 101-102 (1985)

[2] A. Jagota, G. Narasimhan and L. Soltés, A generalization of maximal independent sets, Discrete Applied Math. 109(3): 223-235 (2001)