Bipartite "Slink", an undirected "No Return Game", is the following deterministic game which is solved using theory which you have learned about matchings between girls and boys:

Given a bipartite graph, G, each edge with a girl node and a boy node, and with a pre-chosen girl node, r.
Girls go first.

At each step of the game, the last chosen node, say u, chooses a node, say v, not already chosen and such that (u,v) is an edge of G. The gender of the chosen node which can not choose an unchosen node loses.

Give a polytime algorithm to find structure which insures that the girls can win, or which insures that the boys can win.
Hints. The girls can be sure of winning when every largest matching in G hits node r. The boys can be sure of winning when there is a largest matching in G which does not hit node r.

The DAG No Return Game

For years I used **Slink** as an application while teaching matching theory. My friend Vladimir Gurvich tells me he thinks the game is not known to game theorists, but they might quite like it because the following somewhat similar game is well-know and extensively treated in game theory:

Given a directed acyclic graph G, with a pre-chosen node, r. Two players take turns.

At each step of the game, the player at the latest chosen node, say u, chooses a node, say v, not already chosen and such (u,v) is a directed edge of G. The player at the last chosen node, who can not choose an unchosen node, loses.

Vlad observed that in general, where the directed graph G is allowed to have directed cycles, it is NP complete to decide which player has the sure win.

Slink can be regarded as a special case of having directed cycles. Vlad and I are looking at broader classes of well-solvable No Return games.

Mean Payoff Games, i.e., Cyclic Games

Vlad and several of his Russian friends discovered and found a
wonderful theory of what they called **"cyclic games"**, which are
now famously called **"Mean Payoff Games"**.

Given a directed bipartite graph G, with cycles, with each edge
directed from a girl to a boy or from a boy to a girl, with a given
payoff number, p(e), for each edge e,
and with a pre-chosen node, r.

The **mean payoff**, p(D), of a directed cycle D in G means the sum
of the payoffs of the edges of D divided its number of edges.

At each step of the game, the latest chosen node, say u,
chooses a directed edge (u,v) of G, thereby choosing the node v.
As soon as the chosen edges contain a directed cycle, say D,
the girls win the amount p(D) from the boys.
Equivalently, the boys win the amount -p(D) from the girls.

Central to solving the Mean Payoff Game is the **Payoff Theorem**:

The minimum of p(D) for a cycle D in a subgraph, G″ , of G
which has all edges, (u,v), where u is a girl,
and one edge, (u,v), for each boy u, equals
the maximum of p(D) for a cycle D in a subgraph, G' , of G
which has all edges, (u,v), where u is a boy,
and one edge, (u,v), for each girl u.

A way for the girls to play in order to be sure of winning as much
as this min-max amount is, for a p(D)-maximizing G', to choose,
when at a node u, the edge (u,v) from G'.

A way for the boys to play in order to be sure of losing no more
than this min-max amount is, for a p(D)-minimizing G″, to choose,
when at a node u, the edge (u,v) from G″.

A **"Winner-Take-All Cyclic Game"** is simply to say that the girls win if the achieved p(D) is positive, and the boys win if the achieved p(D) is non-positive.

Then the **Payoff Theorem** says:

Either there is a subgraph G", obtained from G by deleting,
for each boy node, u, all but one edge (u,v),
such that the smallest p(D) for a directed cycle D in G" is non-positive,
or else there is a subgraph G', obtained from G by deleting,
for each girl node, u, all but one edge (u,v),
such that the largest p(D) for a directed cycle D in G' is positive.
(Not both.)

In the first case, the boys can be sure of winning no matter what the girls do.
In the second case, the girls can be sure of winning no matter what the boys do.

Clearly, the Payoff Theorem shows that
the predicate, "the girls can win", is in NP ∩ coNP.